

Department of Higher Education
University of Computer Studies, Yangon
Second Year(B.C.Sc./B.C.Tech.)
Final Examination
Mathematics of Computing II (CST-202)
September, 2018

Answer ALL Questions.

Time allowed: 3 hours

1(a) (i) Find the Laplace transform of $\frac{1}{3}(e^t - 1)^3 e^{-5t}$.

(ii) Find the inverse transform of $(se^{-\pi s/2} + e^{-\pi s})/(s^2 + 1)$.

(b) Solve $y'' - 4y' + 4y = 0$, $y(0) = 8.1$, $y'(0) = 3.9$ using the Laplace transform.

(c) Using the Laplace transform and showing the details, solve

$$y'' + 9y = 8 \sin t \quad \text{if } 0 < t < \pi \text{ and } 0 \text{ if } t > \pi; \quad y(0) = 0, \quad y'(0) = 4.$$

(25 marks)

2(a) Does the sequence $\{a_n\}$ converge or diverge, where $a_n = \frac{8^n}{n!}$? If it converges, find the limit.

(b) Determine whether the following series converge or diverge.

(i) $\sum_{n=1}^{\infty} \frac{8}{\left(3 + \left(\frac{1}{n}\right)\right)^{2n}}$ (Use Root test)

(ii) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{n3^{n-1}}$ (Use Ratio test)

(c) Write out the first five terms of the series, $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n+1}}{n+1}$. Then find the sum of the series.

(20 marks)

3(a) (i) Find the series' radius and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{12^n}$. For what values of x does the series converge absolutely?

(ii) Find the Taylor series and the Taylor polynomials generated by $f(x) = 1/x$, at $a = 2$.

(b) (i) Find the length of the curve, $x = \frac{(2t+3)^{3/2}}{3}$, $y = t + \frac{t^2}{2}$, $0 \leq t \leq 3$.

(ii) Find the Cartesian equation with a polar equation for $4r^2 \cos^2 \theta + 9r^2 \sin^2 \theta = 36$. Then identify the graph.

(20 marks)

4(a) Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$.

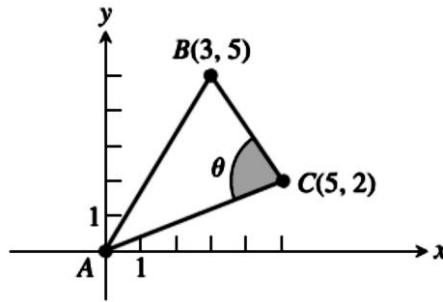
(b) Show that the equation $2x^2 + 3y^2 - 8x - 18y + 29 = 0$ represents an Ellipse. Find its center, foci, vertices and eccentricity.

(c) Find the hyperbola's standard form equation in Cartesian Coordinates:

$$\text{Foci: } (0, \pm\sqrt{10}), \text{ Eccentricity: } \sqrt{5}, \text{ Directrices: } y = \pm \frac{2}{\sqrt{10}}.$$

(20 marks)

5(a) Find the angle θ in the triangle ABC determined by the vertices $A = (0, 0)$, $B = (3, 5)$, and $C = (5, 2)$ shown in the figure.



(b) Find $v \cdot u$, $|v|$, $|u|$, the cosine of the angle between v and u , the scalar component of u in the direction of v , the vector $\text{proj}_v u$. the length and direction (when defined) of $u \times v$ and $v \times u$.

$$v = 9i - 2j + 6k, u = 2i + 2j + k$$

(c) Find the area of the triangle with vertices $P(3, -1, 1)$, $Q(2, 2, -1)$, and $R(-1, 2, 3)$.

(15 marks)
